

An Attack on the Origin of Lift of an Elongated Body

by

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A satisfactory explanation of the mechanism and distribution of the lift forces on a body without sharp edges has never been proposed. Previously, experimental programs have always been necessary to determine the magnitude of the lift and moment for any particular body of the form of airship hulls or of fuselages. Approximate theories proposed have assumed meridional distribution of vortices and have been unsatisfactory although very courageous in view of the fact that the concept of the true flow picture on the surface or in the wake of such bodies was unknown. It is apparent that the lift is not a linear function of α , the angle of attack, nor of $\sin \alpha$ or $\sin 2 \alpha$, but varies with different bodies with respect to Reynolds' Number, fineness ratio, α , and shape.

It was proposed by Professor v. Kármán that an investigation be undertaken to determine the true distribution of vorticity in the wake of various models with view to information which would possibly lead to a rational solution of the lift problem. Hence the results set forth in the present paper are perhaps only a start toward the final goal, since, due to the vast amount of experimental work and computations involved, the investigation was confined to one model.

Airships do not generally fly at very large angles of attack but frequently utilize considerable positive or negative hull lifts to offset weight or buoyancy changes during flight. When such lift is generated the flow in the neighborhood of the fins and control surfaces is quite different from the potential flow pattern. The knowledge of the type of induced flow would be of considerable worth.

The lift derived from fuselages of airplanes is relatively of small magnitude. However, the effect of the side forces on the lateral stability, further the effect of the flow induced by the fuselage on the tail surfaces and the pitching moment of the fuselage may be appreciable.

The present investigation included a survey with a yawmeter of various planes in the wake of an ellipsoid of revolution having a fineness ratio, $F. R.$, of 6. The various conditions involved in the wake measurements were obtained by varying the angle of attack, the wind speed, and the distance from the model to the plane. The flow on the surface of the model was investigated by lampblack and kerosene traces. Force measurements from -4° to 38° and pressure distribution tests were also conducted. The latter tests will be reported in a later paper.

The results of the tests show quite conclusively that the vorticity, and therefore the lift, originated on the rear, leeward side of the body. An increase in velocity tended to move the c. p. position forward as did an increase in angle of attack beyond a certain value. A lift calculation from the vorticity in the wake was made in each case to check the comprehensiveness of the tests. In addition, a simple comparison of C_L , C_M , and c.p. position was made for ellipsoids of fineness ratios of 4 and 6 and for a model of the airship AKRON.

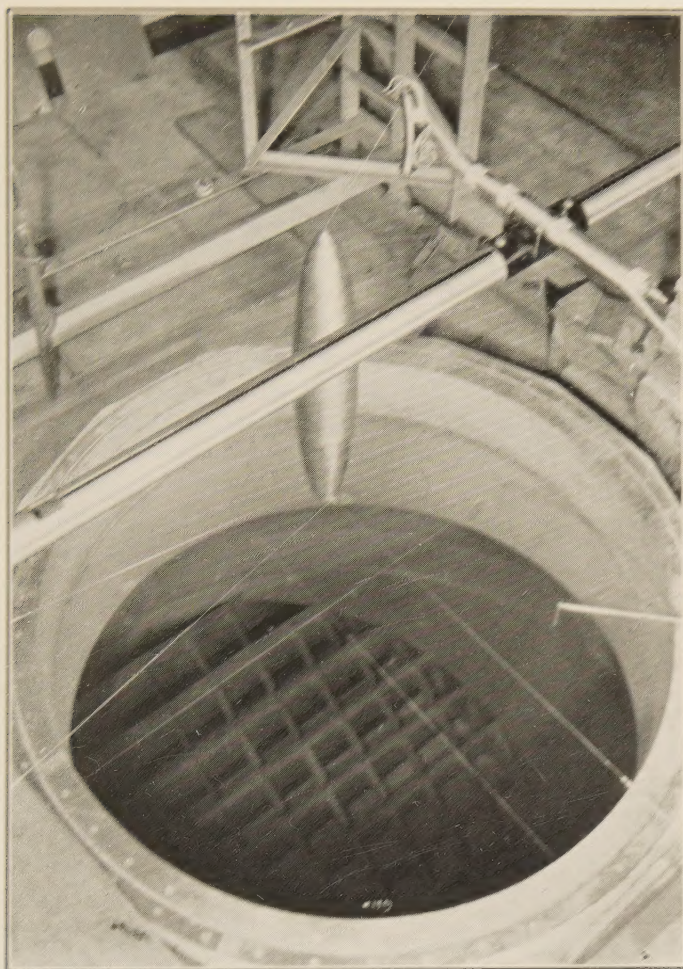


Fig. 1

Model suspension for Wake Investigation.

The procedure followed in the tests will be described in this order: (1) Force measurements, (2) wake investigations, and (3) the development of lampblack and kerosene traces.

Force Tests. The method of suspension of the model for the force measurements was in accordance with the procedure outlined in the first publication of the Daniel Guggenheim Airship Institute. The force readings are considered accurate to 4 grams, corresponding to a lift or drag coefficient increment of 0.0024.

In order to determine the magnitude of the tare lift and drag, a hollow dummy model was used. The dummy had ellipsoidal ends but a cylindrical midsection and was fastened rigidly in its correct position in the tunnel. A rod extended longitudinally through the dummy model and the fittings of the true model were attached to its ends. A weight equal to that of the true model was attached to the rod inside the hollow dummy so that the original tension would be maintained in the balance wires. The rod and weight rode freely in the hollow model and the tare forces were then measured with the same flow interferences that were in evidence in the gross lift and drag measurements.

In order to see if there was any tendency for the model to stall, the measurements were run at increments of 2 to 5 degrees from -5 degrees to as high as 38 degrees, the limit of the suspension system. The velocities were approximately 22.3, 35.7, and 47 meters per second (50, 80, and 105 m.p.h.) which correspond to Reynolds' numbers of 3.6, 5.72, and 7.63×10^5 .

The following pages will first outline the experimental procedure; secondly the results will be discussed; and lastly conclusions will be presented.

EXPERIMENTAL PROCEDURE

The model used in the tests, as mentioned before, was an ellipsoid of revolution with a fineness ratio, or a length/diameter ratio, of six. It was 99 cm. long and 16.5 cm. in diameter at its midsection. To facilitate the connection of tubing for the pressure distribution tests the model was made of two hollow steel castings divided along a meridional plane. The two halves were bolted together and the recesses for the bolt heads were filled to the model surface with wax. Very small fittings were attached at the bow and stern to which the suspension rigging was fastened.

The physical characteristics of the model are summarized in the following table.

TABLE I.

Characteristics of Ellipsoidal Model

Length	0.990 m
Maximum Diameter ..	0.165 m
Fineness Ratio	6.0
Volume	0.01411 m ³
(Volume) ^{1/3}	0.2420 m
(Volume) ^{2/3}	0.0584 m ²

Wake Measurements. The suspension for the wake investigation was fundamentally the same as that for the force tests. Instead of a single vertical drag wire, however, two wires inclined at an angle of about 60 degrees were used to facilitate measurements immediately behind the model.

The instrument selected for these wake surveys was similar to the N.P.L. type yawmeter described in R. and M. No. 844 by Lavender. It is shown quite clearly in Figures 1 and 2 mounted in position behind the model. It employs the null method of direction indication. The operation consists of balancing the pressures exerted on four 0.023 inch hypodermic tubes which are located in sets of two in planes 90 degrees apart, and are all inclined to a common axis at 45 degrees. After the balancing process, which is accomplished from outside of the tunnel, is completed, the angles of the flow direction in the two planes may be read from appropriate protractors from outside of the tunnel airstream. A fifth tube which ends abruptly behind a hollow cone, through which the other four tubes project, gives a pressure reading from which the static pressure may be computed. The static pressure reading was of interest only from the viewpoint of obtaining a complete picture of the wake characteristics. The velocity of the flow can be read from suitable calibration curves. Hence for any point in the wake the inclination of the stream in two planes, the velocity, and the static pressure were obtained simultaneously.

This type of instrument was chosen in preference to the sphere type since it averaged indicated directions over 0.05 inch, which could not be possible with the latter type. This feature proved to be important since the rate of change of direction was found to be as great as 10.5° per inch. The order of accuracy of the measurements was 0.5% for velocity and 0.1° for direction.

The yawmeter was mounted on a tube extending across the airstream which tube was in turn attached to two parallel vertical tubes. It could be moved to any position across the tube by means of a screw arrangement, operated externally and the tube itself could be moved at right angles by a similar drive on the vertical supports. The distance behind the model could be adjusted by raising or lowering the tube and instrument.

The flow in the working section of the tunnel was investigated with the model removed in regard to direction and velocity variations and was found to be in good order with deviations considered to be negligible as compared with expected experimental errors.

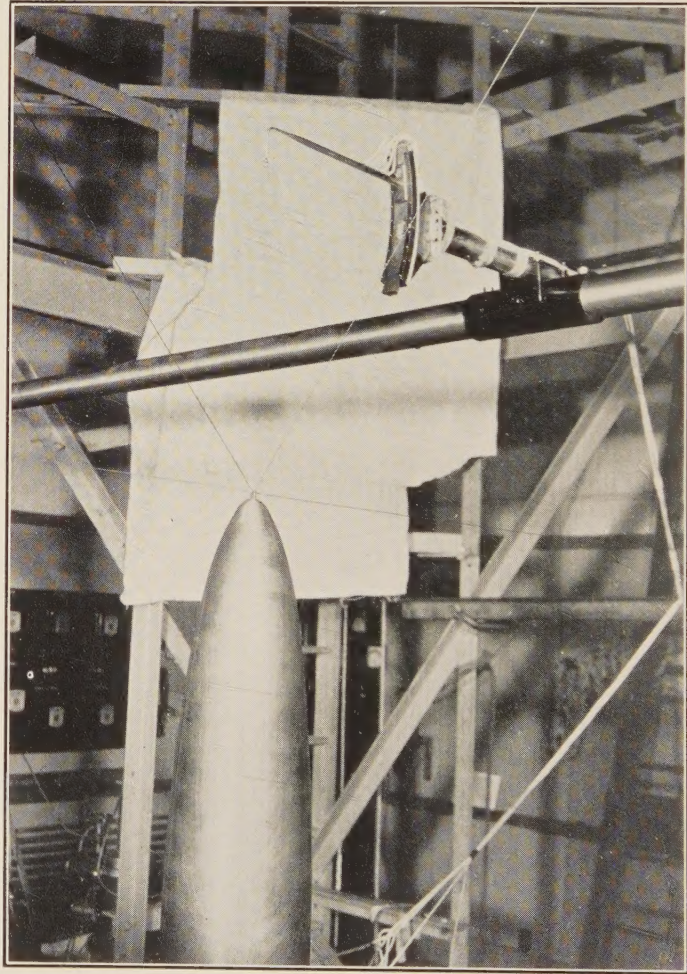


Fig. 2

NPL instrument behind model.

The experimental procedure was to traverse various planes in the wake of the model and perpendicular to the undisturbed flow, U_0 . The various angles of attack, velocities, and locations of the planes are summarized in table IV. The distance of the planes from the stern of the model was measured parallel to U_0 . The area of the planes traversed with the instrument extended until the direction of the flow deviated less than 0.1° from that of U_0 . The largest area was 75 cm. long and 50 cm. wide.

The distance between points of measurement varied from 0.5 cm. to 5 cm. depending on the direction gradients at the particular locality. For the complete survey the flow characteristics at more than 4,500 points were investigated.

Lampblack and Kerosene Tracing. During the time the model was in the tunnel it was painted lightly with a dilute solution of lampblack in kerosene. The tunnel was immediately run at the desired speed until the mixture had dried on the model. Photographs of the model showing the traces of the flow on the windward, leeward, and side surfaces were taken. To show the traces clearly, the photographs were made in three parts for each view of the model and consequently do not match entirely satisfactorily since the model could not be removed from the tunnel at the time.

Explorations of the model surface were made with a short silk thread to determine the correctness of the traces. It was discovered that the traces corresponded exactly to the direction indicated by the exploring thread with the exception of a small area adjoining the aft suspension fixture where it was impossible to decide whether or not this was true due to excessive turbulence. Photographs of the traces for one particular test are included.

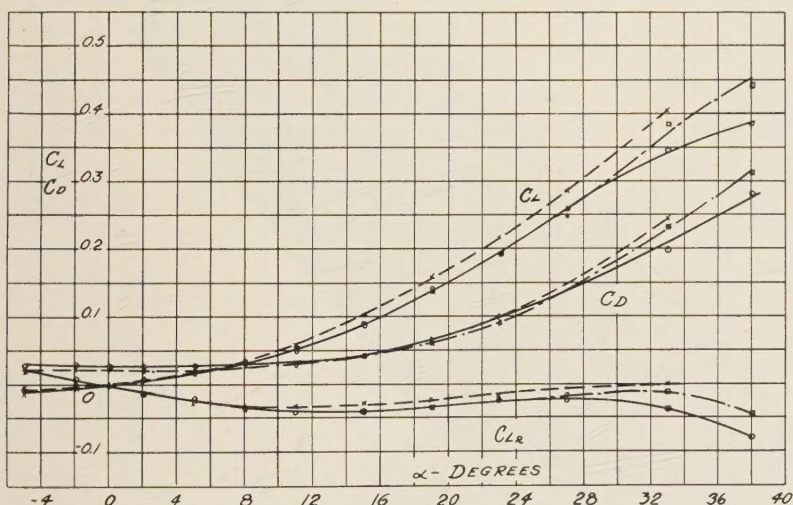


Fig. 3

- $R_e = 3.6 \times 10^5$
- $R_e = 5.72 \times 10^5$
- × $R_e = 7.63 \times 10^5$

Lift, Drag, and Moment Curves for Ellipsoid, F.R. = 6.

values of R_e coincide while that for the highest value of R_e is distinctly above, but at higher angles of attack all three curves are quite separate.

The drag curves for the highest and lowest R_e values coincide up to about 24° while that for the middle value of R_e is lower. By referring to Troller's paper in the first publication of the Daniel Guggenheim Airship Institute, it may be seen that a minimum drag coefficient is obtained for the short airship model in the Akron tunnel at a value of R_e equal to 5×10^5 which is close to the present value of R_e equal to 5.72×10^5 for lowest C_D .

Values of C_{L_R} , coefficient for the rear lift component, are also given. These curves will enter into a later discussion (p. 35). It is interesting to note their general trends. The values for C_L and C_D are tabulated in Table II.

DISCUSSION OF RESULTS

The order of the discussion will be the same as for the presentation of the experimental procedure with an additional note on the center of action of the lift vortices.

Force Measurements. The lift and drag curves for the tests run at the three values of R_e mentioned previously, are shown in Figure 3. It is evident that the angle at which stalling occurs is being approached near the highest investigated values of α . Up to 28° the C_L curves for the two lower

TABLE II.

Re α	C_L and C_D FOR ELLIPSOID, FINENESS RATIO = 6.					
	3.6×10^5		5.72×10^5		7.63×10^5	
	C_L	C_D	C_L	C_D	C_L	C_D
-5°	-0.0123	0.0305	-0.0118	0.0245	-0.0096	0.0228
-2°	-0.0029	0.0278	-0.0046	0.02535	-0.00304	0.02536
0	-0.0006	0.0261	+0.00124	0.0247	+0.0023	0.02346
2	+0.0061	0.0274	0.00515	0.0265	0.0073	0.02364
5	0.0214	0.02845	0.0166	0.0249	0.0186	0.0252
8	0.0322	0.0310	0.0312	0.0259	0.0349	0.02675
11	0.0496	0.0338	0.0526	0.0306	0.0599	0.03385
15	0.0867	0.0431	0.0890	0.0410	0.1044	0.0434
19	0.1415	0.0673	0.1381	0.0602	0.1596	0.0670
23	0.1920	0.1025	0.1930	0.09555	0.2159	0.1011
27	0.2594	0.1405	0.2486	0.1329	0.2867	0.1454
33	0.3446	0.1998	0.3857	0.2344	0.4056	0.2455
38	0.3851	0.2810	0.4411	0.3125		

Lampblack Photographs and
Wake Investigation.

Stern

For the purpose of locating the various planes or points in planes, the following notation will be used. If the origin of the coordinate system be considered to be at the aft end of the major axis of the ellipsoid, the positive x axis will extend astern parallel to the undisturbed flow. The z axis will be considered as positive upward and extending in the pitching



Bow

Fig. 4

plane in a direction perpendicular to the undisturbed flow, and the y axis will be perpendicular to the xz plane, positive to the starboard side. The velocity components, v and w , will be parallel to y and z respectively.

Assuming, at first, that the source of vorticity is not obvious, we may consider the mechanics of the origin of the vorticity. Referring to Figure 4, which shows the windward side of the ellipsoid at 21.5°

and at an R_e of 5.72×10^5 corresponding to a velocity of 35.6 m/s (80 m.p.h.), the traces made by lampblack and kerosene depict a flow which would be expected from considerations of the potential theory. In general, the flow divides at the meridian in the pitching plane and moves along the surface of the ellipsoid in a path such as the intersection of a curved surface with the contour of the ellipsoid.

If potential flow did not break down, we would expect that the traces on the leeward side of the model would be identical with those shown on the windward side if the photograph were reversed so that the stern of the windward side coincided with the bow of the leeward side. Figure 5 shows that the duplication of traces is quite close in this comparison for the forward part of the leeward side, but for the rear section the flow is shown to deviate decidedly from the potential flow. The traces commence to "feather" out from the leeward meridian and continue around the ellipsoid to join tangentially those traces extending from the windward side. This line of intersection of the traces clearly coincides with the line of minimum pressure along the ellipsoid.

Apparently, in working against an adverse pressure gradient along the leeward side, the energy of the flow in the boundary layer has decreased due to frictional effects until the flow is forced to take a path of lesser resistance which is outward toward the minimum pressure region as indicated by the traces. Consequently, the pressure is not built up in the stern of the leeward side according to potential theory and the net result is a lift on the ellipsoid.



Fig. 5

More striking is the fact that while the authenticity of the traces was being investigated with the exploring thread, it was discovered that the flow in this same region at a little less than 1 cm. from the surface was in the direction which would be expected if the flow were potential. In other words, the thread showed that the flow "feathered" inward toward the leeward meridian in approximately the same manner that the flow immediately upon the surface "feathered" outward. A surface of discontinuity apparently existed in this locality a short distance from the

surface. The direction of the flow on either side of this surface was such that vorticity would be originated there with rotation in the direction found in the wake measurements.

The traces in Figure 5 show that the "feathering" out commences mildly at about 15 cm. ahead of the center of buoyancy and is confined to a narrow band of about 5 cm. width. The most decided "feathering" commences 20 cm. aft of the center of buoyancy and is as wide as 9 cm. gradually narrowing down as the stern is approached.

In Figure 6 the traces along the side of the ellipsoid are shown. The flow from the windward and leeward sides intersecting along the minimum pressure line is depicted more clearly. The curvature and pattern of the traces suggests a flow which is a combination of that around a cylinder with that about an airfoil.

The traces on the ellipsoid (not shown here) at an angle of pitch of 28° show that the discontinuous region extends farther forward than for the angle of attack of 21.5° . Comparing traces at two different speeds for the same angle one sees that the "feathering" region is farther forward for the higher speed than for the lower speed. Deductions bearing out these statements will be presented later.

From the data taken during the wake surveys, the velocity components v and w , perpendicular to U_0 , were computed. These components were plotted as a vector field. Figures 7 to 15 inclusive show these vector fields. Hereafter, the planes will be referred to by their letter as designated in Table IV.



Fig. 6

Plane F, where $\alpha = 21.5^\circ$, $x = 20.0$ cm., and $U_0 = 22.3$ m/s, will be discussed in greater detail than the other planes. In order to discover the magnitude of the w components in the wake due to potential flow at 21.5° , values of w/U_0 were calculated for points located in various planes at $y = 0$, $z = 1.0$ cm., the approximate location of maximum actual vertical velocity components. The values are plotted in Figure 16. Since in this investigation the closest plane was at $x = 9.6$ cm., the values of w/U_0 due to potential flow would not affect the actual w components to any great extent.

To give a complete picture of what is happening in the wake curves of constant values of w/U_0 , v/U_0 , and $\frac{(P_{s1} - P_s)}{q_0}$ for plane F were plotted in Figures 17, 18, and 19, respectively, where $(P_{s1} - P_s)$ is the static pressure in the wake referred to that at a great distance ahead of the model and q_0 is the dynamic pressure of the undisturbed flow. These curves indicate that

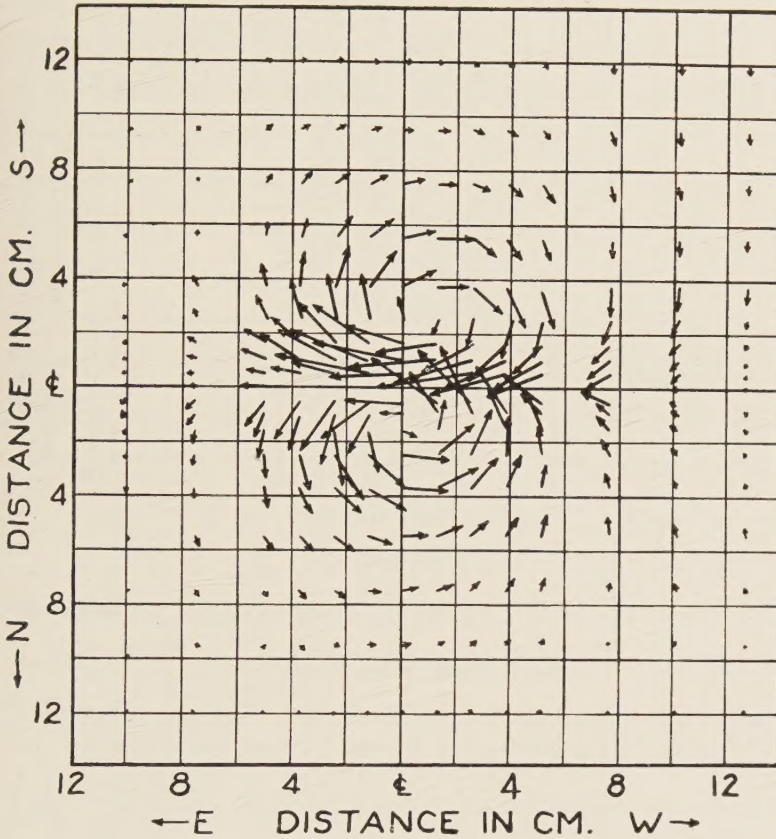


Fig. 7—Plane A.

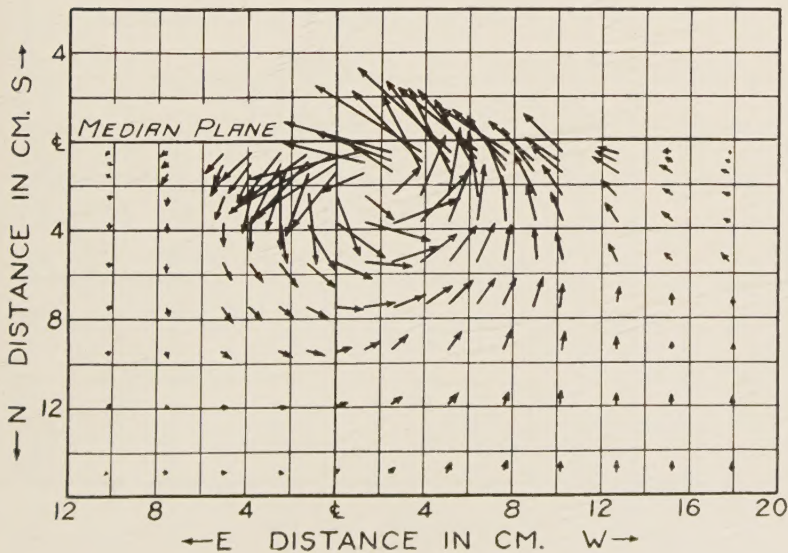


Fig. 8—Plane B.

the maximum value of v/U_0 is approximately 0.12, and that the flow is in the direction indicated on the vector field diagram. The maximum values of w/U_0 occur at the midpoint of a line joining the centers of the two swirls. It has a value of 0.325 and is negative. The maximum positive value of w/U_0 at $y = \pm 8$ cm., $z = 3$ cm., is 0.10.

The region of highest negative static pressure has the value $0.21 q_0$ and is located midway between the two swirls and corresponds to the place of highest induced velocity. The negative pressure region is limited to an oval section about 21 cm. long and 14 cm. wide. Outside of this region, areas of positive pressure exist with a maximum value of $0.04 q_0$ as the curves indicate. These three figures show that the flow in the wake is quite symmetrical about the z axis so in a few cases only one-half of a plane was traversed with the yawmeter.

Plane E, which entails the same flow conditions as in plane F but was located at $x = 9.6$ cm., will be compared with plane F. Evidently in Figures 11 and 12 the vortices generated on the ellipsoid have already rolled up into what approximates

two vortex cores. The dashed line, $y = \text{constant}$, in Figure 11, through the cores and constructed through the ends of the velocity vectors, approximates a distribution similar to that along a line through a vortex core. By comparing the planes it is apparent that the swirls have moved downward from $z = 4.5$ to $z = 2.6$ which is to be expected from theoretical considerations. This again indicates that the vortices originated on the rear leeward surface of the ellipsoid.

Figure 20 shows lines of constant values of local vorticity divided by U_0 , $\frac{1}{U_0} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right)$ which were calculated by taking the tangents to the velocity curves of v and w plotted against z and y respectively. This expression for vorticity assumes a line lifting vortex since the longitudinal component of vorticity could not be determined experimentally. Figure 21 shows these same values of vorticity plotted three

dimensionally as two vortex "mountains." The maximum value of $\frac{1}{U_0} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right)$ is about 12.0, which corresponds to an angular velocity of 134 radians per second. The area of positive vorticity is confined to an approximately circular section of 20 cm. diameter while entirely surrounded by an area of weak negative vorticity with a maximum value of $\frac{1}{U_0} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right)$ equal to -0.5 . It has been proposed that the negative vorticity may be explained in the following manner.

Behind a flat plate parallel to the flow the flow pattern is like that shown in Figure 22 (a). This is of course the remainder of the boundary layer due to skin friction. If such a remainder of the primary boundary layer of the ellipsoid is apparent in the wake, and if the boundary layer flow has a strong inclination to the main flow, U_0 , which it does in this

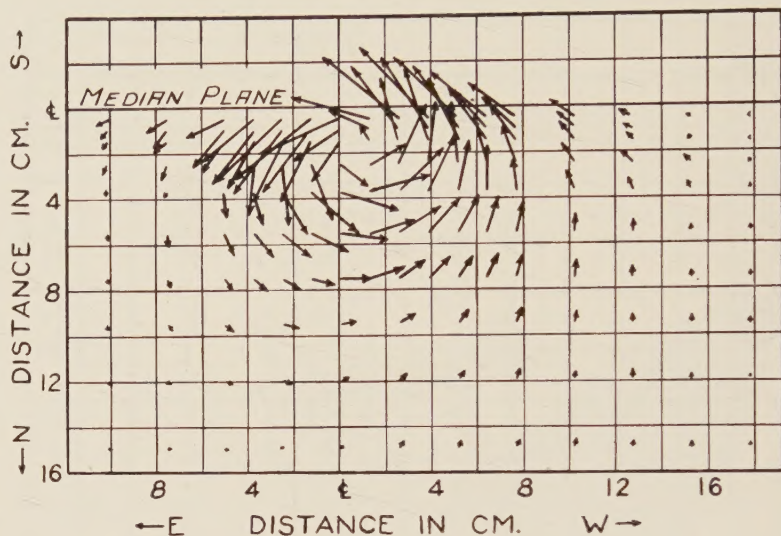


Fig. 9—Plane C.

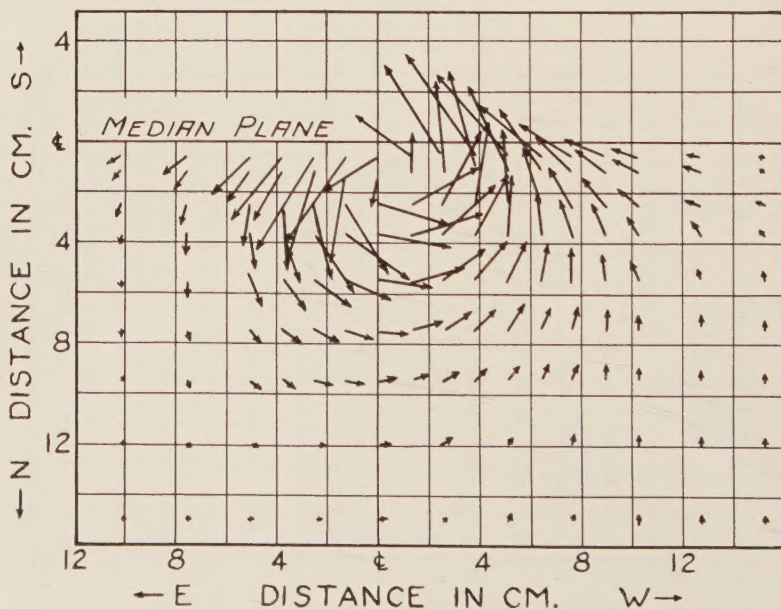


Fig. 10—Plane D.

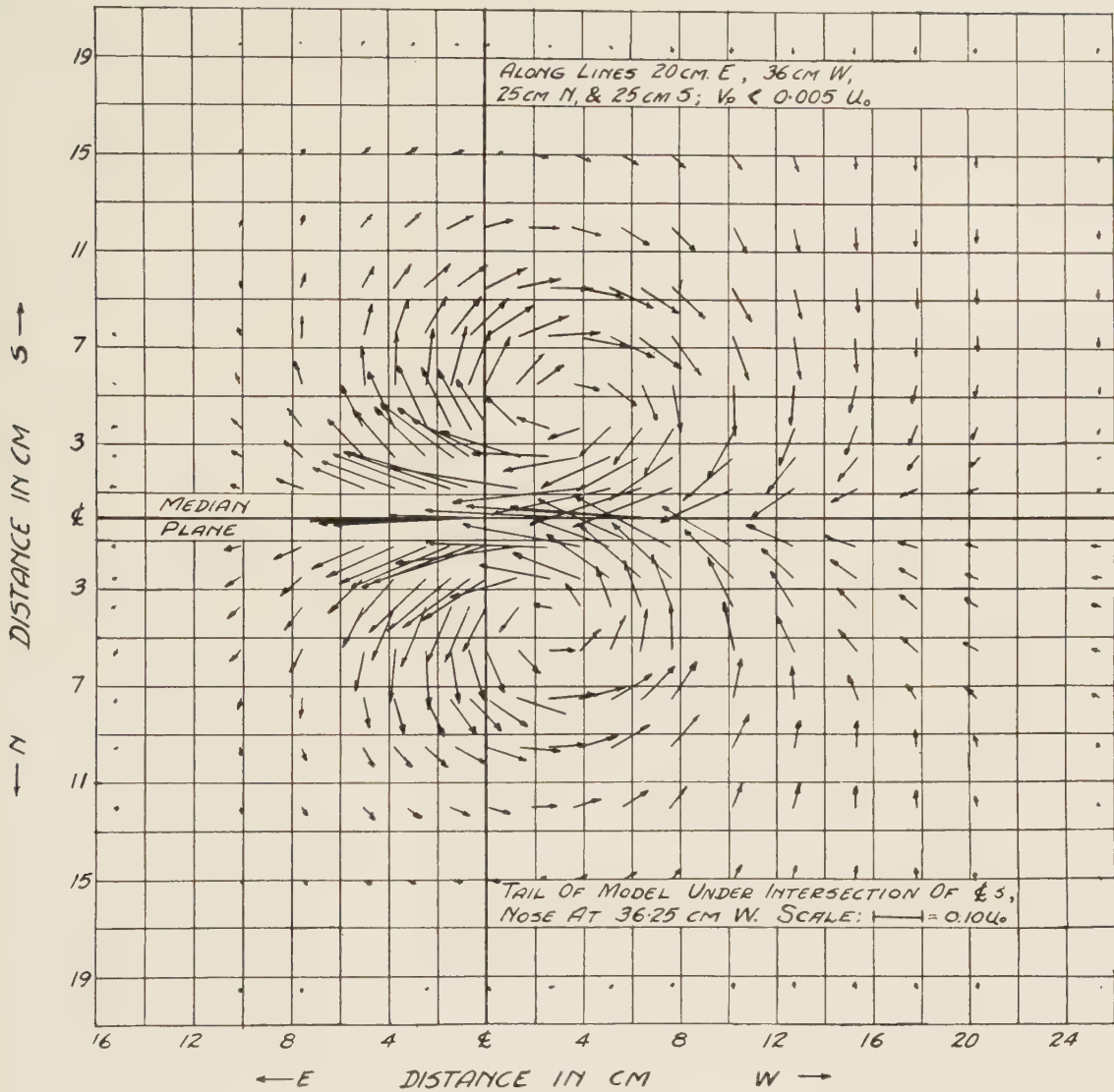


Fig. 11 — Plane E.

case of the inclined ellipsoid, the boundary layer would furnish a vortex component in the plane normal to U_0 . This component is negative. Figure 22 (b) shows the whole system of vortices diagrammatically. The secondary movement or discontinuity boundary layer gives rise to the main lift vortex cores.

Obviously, to calculate the vorticity of the lift vortices to the exclusion of the negative vorticity component of the primary surface boundary layer, the calculation should be made for a plane normal to the actual average direction of flow in the wake. This direction would lie between the x axis and the longitudinal axis of the ellipsoid. Such a calculation will be made in connection with future tests.

A preliminary check on the comprehensiveness of the tests has been made by a calculation of lift on the ellipsoid from the spanwise distribution of circulation, Γ_y , in the wake. Γ_y was determined by integrating the velocity in the various planes around rectangles all having a common side, $y = 0$, and extending out along $z = z_1$ and $z = z_2$ to $y = y_1$, $y = y_2$, etc. according to the equation

$$\Gamma_y = \int (v dy - w dz) \dots \dots \dots (1)$$

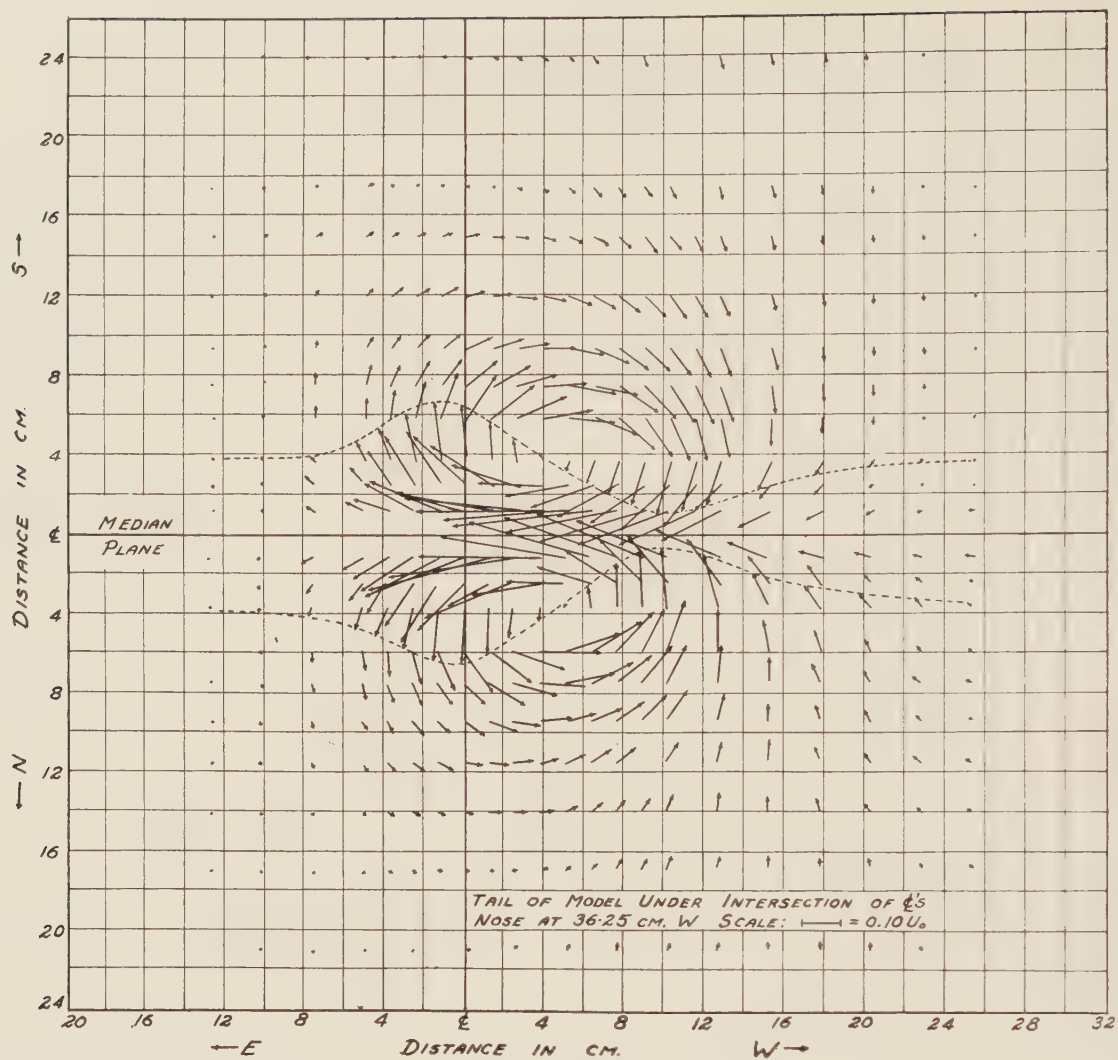


Fig. 12—Plane F.

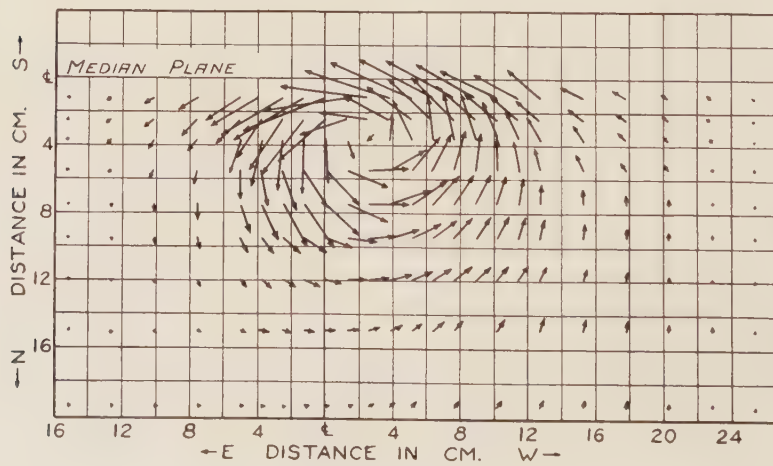


Fig. 13—Plane G.

The lines $z = z_1$, and $z = z_2$ were chosen so as to include as much of the positive vorticity as possible and to exclude the negative vorticity. The effect of the negative vorticity has been felt already by nullifying some of the positive vorticity so the calculated lift is not as great as the lift measured in the force tests.

The lift calculation was made in the following manner. If we assume that

γ = circulation around any element of the span.

γ_0 = circulation around center of span.

b = total hypothetical span.

ρ = mass density of air.

$\Gamma_{(y)}$ = value of trailing vortices between $y = 0$ and $y = y_1$.

then

$$\begin{aligned} L &= \rho U_0 \int_{-b/2}^{+b/2} \gamma dy \\ &= \rho U_0 \int_{-b/2}^{+b/2} (\gamma_0 - \Gamma_{(y)}) dy \\ &= \rho U_0 \gamma_0 b - \rho U_0 \int_{-b/2}^{+b/2} \Gamma_{(y)} dy \dots\dots\dots (2) \end{aligned}$$

This, of course, may also be deduced from the expression

$$L = \rho U_0 \int_{-b/2}^{+b/2} \frac{d\Gamma_{(y)}}{dy} y dy \dots\dots\dots (3)$$

Integrating partially

$$\begin{aligned} L &= \rho U_0 \left[\Gamma_{(y)} y - \int \Gamma_{(y)} dy \right]_{-b/2}^{+b/2} \\ &= \rho U_0 \gamma_0 b - \rho U_0 \int_{-b/2}^{+b/2} \Gamma_{(y)} dy \dots\dots\dots (2) \end{aligned}$$

Figure 23 shows the spanwise distribution of Γ_y/U_0 for plane F calculated from equation (1), and the corresponding values of $\frac{1}{U_0} \cdot \frac{d\Gamma_{(y)}}{dy}$ are shown in Figure 24. The lift was calculated then by means of equation (3). This served as a check on the lift by equation (2).

In order to assure the accuracy of the values of local vorticity, $\frac{1}{U_0} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right)$, these values were integrated along lines $y = y_1$, $y = y_2$, etc., according to the equation

$$\frac{1}{U_0} \cdot \frac{d\Gamma_{(y)}}{dy} = -\frac{1}{U_0} \int \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \cdot dz \text{ and plotted.}$$

Then the lift was calculated by graphical evaluation of the integral

$$L = \rho U_0 \cdot \iint \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) y \cdot dy \cdot dz \dots\dots\dots (4)$$

The results of the various calculations are given in Table III.

TABLE III

Results of Lift Calculations for Plane F, $x = 20$ cm, $U_0 = 22.3$ m/s,
 $\alpha = 21.5^\circ$

C_L from force measurements	0.179
C_L from equation (2)	0.165
C_L from equation (3)	0.166
C_L from equation (4)	0.169

Good agreement is evidenced between the results of the three methods of integration. Equation (4) gives the highest result because it covered only the area of positive vorticity. Agreement with the results of the force tests in all cases was close in view of the fact that negative vorticity was present and that the longitudinal component of vorticity was neglected. Apparently for this plane these two conditions are not strongly felt.

The lift calculation was made for all planes. The complete set of curves used for the calculation will not be included here since they are of the same general shape as those for plane F which have been described. Table IV will show the main characteristics of the wake picture for each plane.

By examination of the values given in the table it is seen that the effect of increasing R_e is in general to increase the value of z for the vortex core centers. This is to be expected as the result of the fact that the discontinuity region spreads forward on the ellipsoid if U_0 is increased as was already seen from the lampblack and kerosene traces. Increasing U_0 increases the values of

$$\left(\frac{\partial \Gamma_{(y)}}{\partial y} \right) \max$$

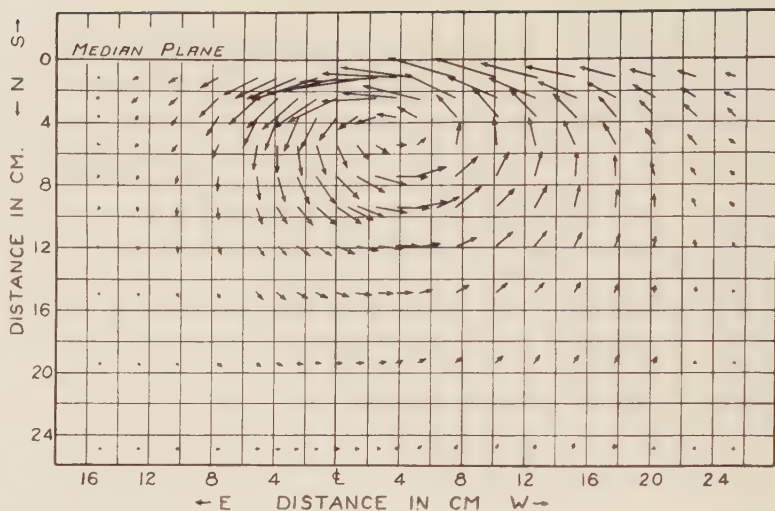


Fig. 14—Plane H.

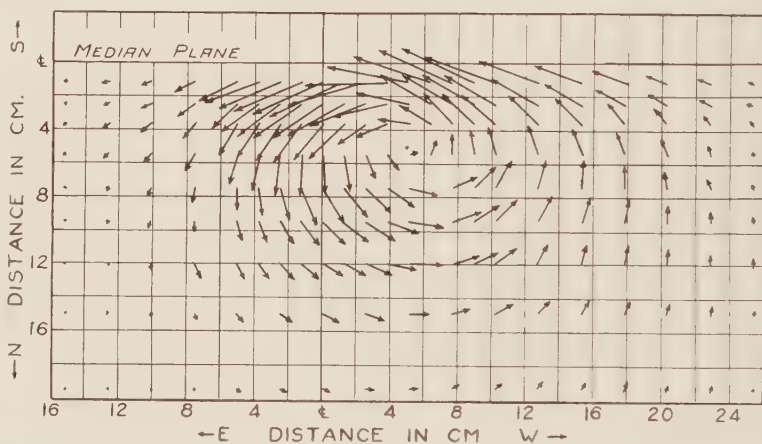


Fig. 15—Plane J.

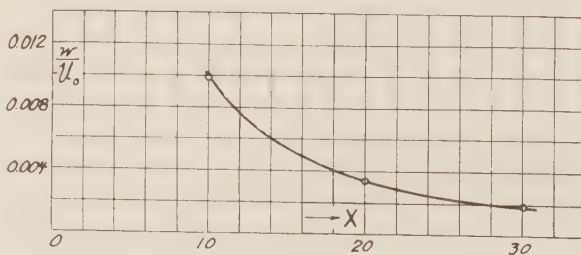


Fig. 16.

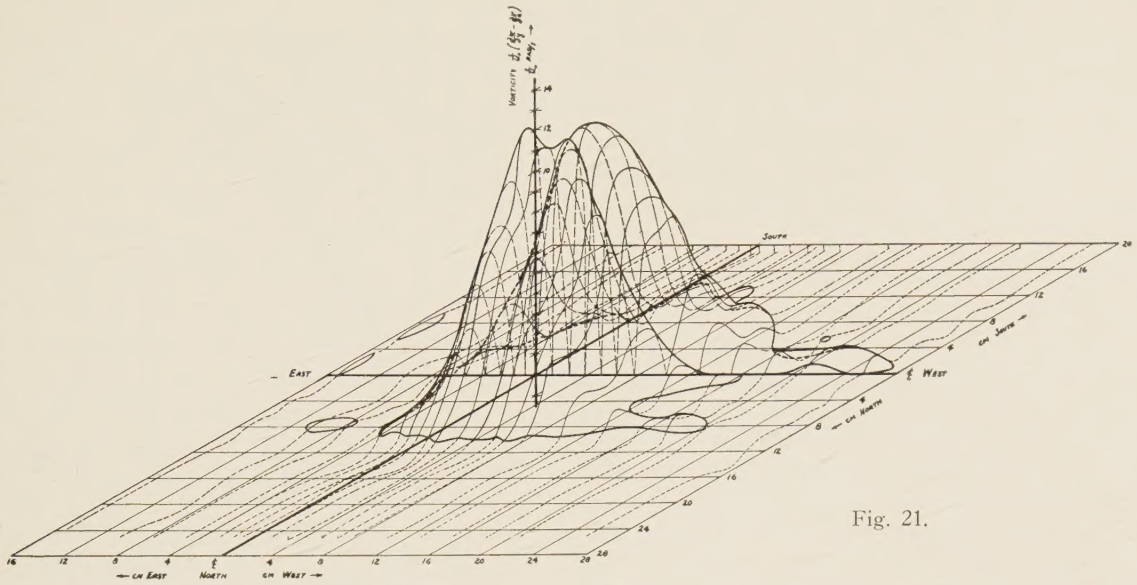


Fig. 21.

The general tendency of the lift vector for the present ellipsoid to have a maximum rearward location at 12° pitch is shown by referring again to Figure 3 where the coefficient of lift based on the rear lift alone is plotted. It is seen that these negative values of C_{LR} are maximum at 12° pitch, and as the angle of pitch is increased they diminish to zero. This shows that the center of lift is apparently moving forward, which fact again agrees with what has been shown previously in this paper.

It is believed that if contours of the difference of the actual pressures and the theoretical pressures are plotted on the leeward side of the ellipsoid, the exact location of the vorticity origin and distribution on the surface will be found. It is the plan to do this as soon as possible.

Additional information could be gained by getting the local vorticity distribution for all of the planes, especially for the planes closer to the model. This is a particularly long and tedious process.

CONCLUSIONS

(1) An attempt has been made to show that the origin of vorticity, and therefore of lift, is confined to a definite area on the rear leeward surface of the model.

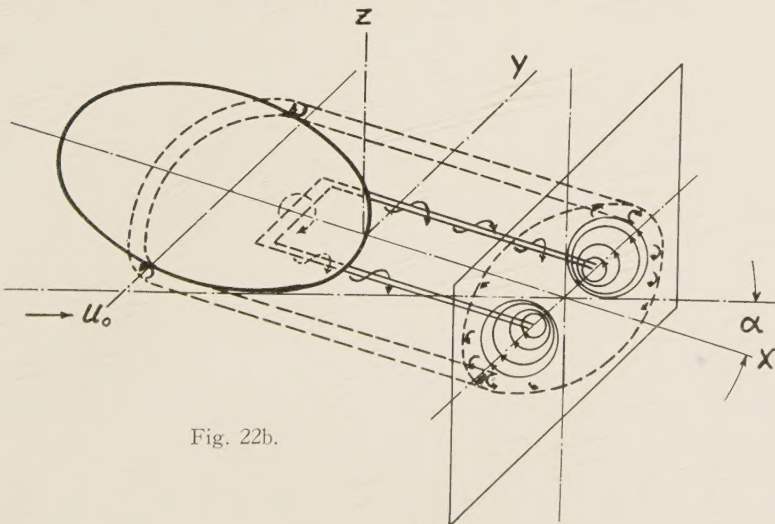


Fig. 22b.

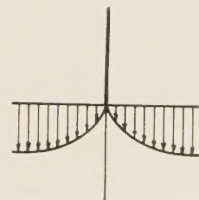


Fig. 22a.

(2) From flow characteristics on and adjacent to the model a surface of discontinuity apparently exists where the direction of the flow is such as to generate vorticity with rotation in the direction indicated by wake measurements. The flow on the windward side of the ellipsoid is similar to that expected in the light of potential theory.

(3) Additional evidence pointing to the generation of vorticity on the rear leeward side is indicated by the distribution of vorticity in the wake. The configuration of the induced velocities in the wake is that of two distinct vortex cores. Lift calculations from the vorticity in the wake are in good agreement with the measured values. The differences were explained by the presence of negative vorticity components of the primary boundary layer of the model and also by the fact that the longitudinal components of the vorticity were necessarily neglected in the plane normal to the direction of undisturbed flow.

(4) An attempt was made to calculate the variation of the position of the lift vector from the difference between the theoretical and actual pitching moments. The results showed a maximum rearward location of the vector at about 12° pitch for the present model. Comparison was made with an ellipsoid of fineness ratio = 4 and an airship model with fineness ratio = 5.92.

(5) It is believed that the negative vorticity component of the boundary layer at angles where strong inclination of the flow is evidenced may cause a reduction in rate of change of lift with angle of pitch, the negative increment of lift being caused by skin friction.

(6) The variation of rear lift substantiates the argument that there is a maximum rearward position of c.p. of the lift vector.

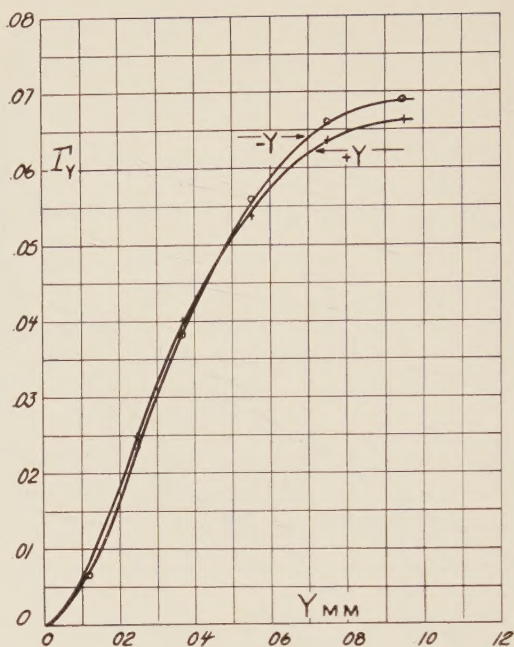


Fig. 23.

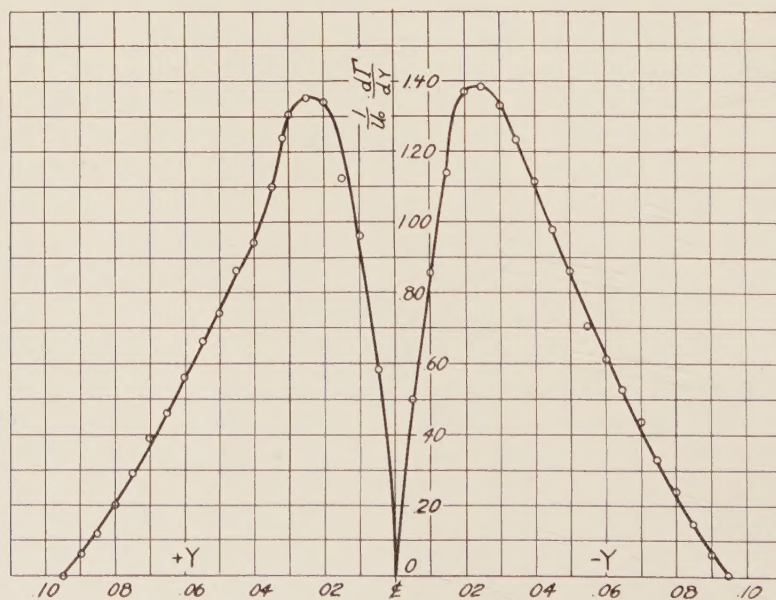


Fig. 24.

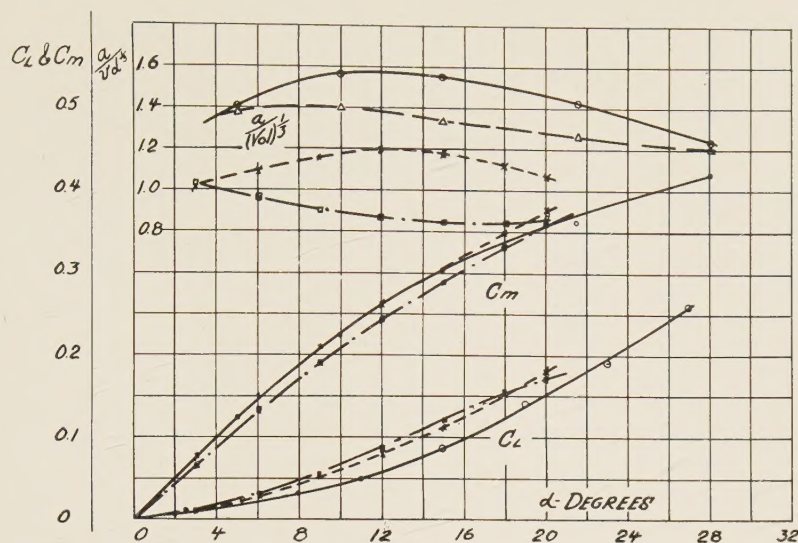


Fig. 25.

○ Ellipsoid	F.R. = 6	$R_e = 360,000$
△ Ellipsoid	F.R. = 6	$R_e = 763,000$
□ Ellipsoid	F.R. = 4	$R_e = 160,000$
× Akron Model	F.R. = 5.92	$R_e = 3,550,000$

It is hoped, when additional computations have been made in regard to eliminating the negative vorticity components, to plotting pressure variations on the surface of the ellipsoid, and to calculating the distribution of lift vorticity in other planes in the wake, that a rational treatment of the lift for elongated bodies will be approached.

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Designations and Symbols Used Frequently:

$R_e =$	$\frac{(Vol)^{1/3} U_o}{\nu}$	Reynolds Number.
(Vol)		Volume of ellipsoid or airship.
U_o		Velocity of undisturbed flow.
ν		Kinematic viscosity.
$C_L =$	$\frac{L}{q (Vol)^{2/3}}$	Lift coefficient.
$C_D =$	$\frac{D}{q (Vol)^{2/3}}$	Drag coefficient.
$C_{LR} =$	$\frac{L_R}{q (Vol)^{2/3}}$	Coefficient of rear lift component.
$C_M =$	$\frac{M}{q (Vol)}$	Moment coefficient.
L		Lift.
D		Drag.
M		Moment.
$q_o =$	$\rho/2U_o^2$	
ρ		Density of the air.
F.R.		Ratio of length to greatest diameter, fineness ratio.
C.P.		Center of pressure.
α		Angle between U_o and greatest axis of ellipsoid or airship.
K_1, K_2		Apparent mass coefficients.